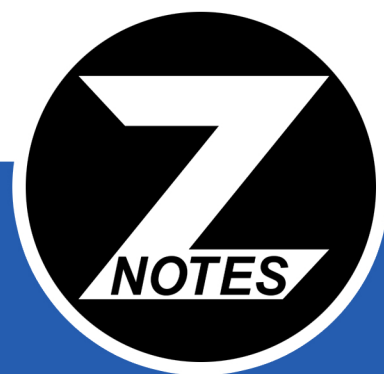




Updated to 2019 Syllabus

CIE A-LEVEL MATHS 9709 (M1)

FORMULAE AND SOLVED QUESTIONS FOR MECHANICS 1 (M1)

TABLE OF CONTENTS

2 | CHAPTER 1
Velocity & Acceleration

3 | CHAPTER 2
Force & Motion

3 | CHAPTER 3
Vertical Motion

3 | CHAPTER 4
Resolving Forces

4 | CHAPTER 5
Friction

4 | CHAPTER 6
Connected Particles

7 | CHAPTER 7
Work, Energy & Power

7 | CHAPTER 8
General Motion in a Straight Line

1. VELOCITY AND ACCELERATION

1.1 Kinematics Equations

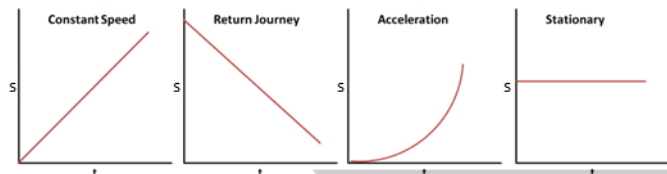
$$v = u + at$$

$$s = ut + \frac{1}{2}at^2 \text{ and } s = vt - \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u + v)t$$

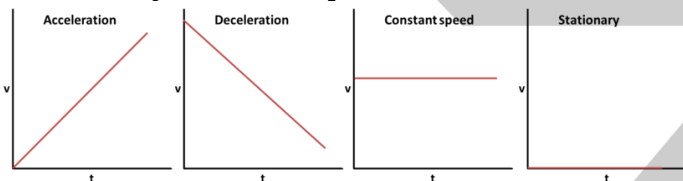
$$v^2 = u^2 + 2as$$

1.2 Displacement-Time Graph



- Gradient = speed

1.3 Velocity-Time Graph



- Gradient = acceleration
- Area under graph = change in displacement

{S12-P42}

Question 7:



The small block has mass 0.15kg . The surface is horizontal. The frictional force acting on it is 0.12N . Block set in motion from X with speed 3ms^{-1} . It hits vertical surface at Y 2s later. Block rebounds from wall directly towards X and stops at Z . The instant that block hits wall it loses 0.072J of its kinetic energy. The velocity of the block from X to Y direction is $v\text{ms}^{-1}$ at time $t\text{ s}$ after it leaves X .

- Find values of v when the block arrives at Y and when it leaves Y . Also find t when block comes to rest at Z . Then sketch a velocity-time graph of the motion of the small block.
- Displacement of block from X , in the $\overrightarrow{XY}$ direction is $s\text{ m}$ at time $t\text{ s}$. Sketch a displacement-time graph. On graph show values of s and t when block at Y and when it comes to rest at Z .

Solution:

Part (i)

Calculating deceleration using Newton's second law:

$$0.12 = 0.15a \quad a = \frac{0.12}{0.15} = 0.8\text{ms}^{-2}$$

Calculate v at Y using relevant kinematics equation

$$-0.8 = \frac{v-3}{2} \quad v = 1.4\text{ms}^{-1}$$

Calculate kinetic energy at Y

$$E_K = \frac{1}{2}(0.15)(1.4)^2 = 0.147\text{J}$$

Calculate energy lost:

$$\text{Initial} - \text{Change} = \text{Final}$$

$$0.147 - 0.072 = 0.075\text{J}$$

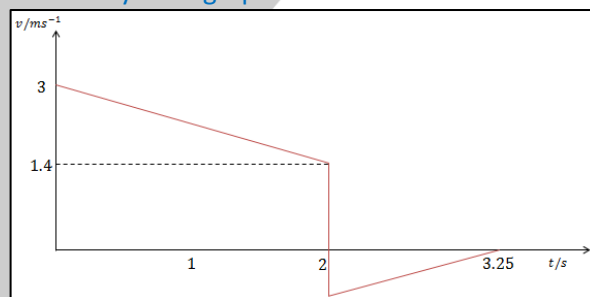
Calculate speed as leaving Y using $k. E.$ formula:

$$0.075 = \frac{1}{2}(0.15)v^2 \quad v = 1\text{ms}^{-1}$$

Calculate t when particle comes to rest:

$$-0.8 = \frac{0-1}{t} \quad t = 1.25\text{s}$$

Draw velocity-time graph with data calculated:



Part (ii)

Calculate displacement from X to Y

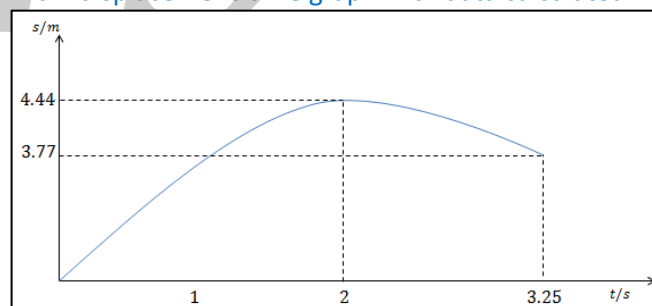
$$s = (3 \times 2) + \frac{1}{2}(-0.8)(2)^2 \quad s = 4.4\text{m}$$

Calculate displacement from Y to Z

$$s = (1 \times 1.25) + \frac{1}{2}(-0.8)(1.25)^2$$

$$s = 0.625\text{m in the opposite direction}$$

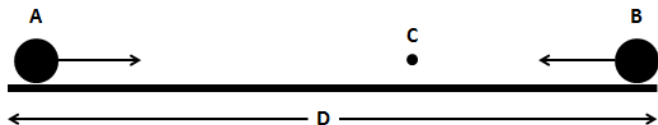
Draw displacement-time graph with data calculated:



1.4 Average Velocity

- For an object moving with constant acceleration over a period of time, these quantities are equal:
 - The average velocity
 - The mean of initial & final velocities
 - Velocity when half the time has passed

1.5 Relative Velocities



- Let s_A be the distance travelled by A and s_B for B

$$s_A = ut + \frac{1}{2}at^2 \quad s_B = ut + \frac{1}{2}at^2$$
- If a collision occurs at point C

$$s_A + s_B = D$$
- This gives you the time of when the collision occurred
- Same analysis if motion is vertical

2. FORCE AND MOTION

Newton's 1st Law of Motion:

Object remains at rest or moves with constant velocity unless an external force is applied

Newton's 2nd Law of Motion:

$$F = ma$$

3. VERTICAL MOTION

- Weight:** directly downwards
- Normal contact force:** perpendicular to place of contact

3.1 Common Results of Vertical Motion

Finding time taken to reach maximum height by a projectile travelling in vertical motion:

$$v = u + at$$

- Let $v = 0$ and find t
- The time taken to go up and come back to original position would be double of this t

Finding maximum height above a launch point use:

- $2as = v^2 - u^2$
- Let $v = 0$ and find s

Finding time interval for which a particle is above a given height:

- Let the height be H and use
- $s = ut + \frac{1}{2}at^2$
- Let $s = H$
- There will be a quadratic equation in t
- Solve and find the difference between the 2 t 's to find the time interval

{S04-P04}

Question 7:

Particle P_1 projected vertically upwards, from horizontal ground, with speed 30ms^{-1} . At same instant P_2 projected vertically upwards from tower height 25m , with speed 10ms^{-1}

- Find the time for which P_1 is higher than the top of the tower
- Find velocities of the particles at instant when they are same height
- Find the time for which P_1 is higher than P_2 and moving upwards

Solution:

Part (i)

Substitute given values into displacement equation:

$$25 = (30)t + \frac{1}{2}(10)t^2$$

$$5t^2 + 30t - 25 = 0$$

Solve quadratic for t

$$t = 1\text{s or } 5\text{s}$$

P_1 reaches tower at $t = 1$ then passes it again when coming down at $t = 5\text{s}$

Therefore time above tower = $5 - 1 = 4$ seconds

Part (ii)

Displacement of P_1 is S_1 , and of P_2 is S_2 & relationship:

$$S_1 = 25 + S_2$$

Create equations for S_1 and S_2

$$S_1 = 30t + \frac{1}{2}(-10)t^2 \quad S_2 = 10t + \frac{1}{2}(-10)t^2$$

Substitute back into initial equation

$$30t + \frac{1}{2}(-10)t^2 = 25 + 10t + \frac{1}{2}(-10)t^2$$

Simple cancelling

$$t = 1.25\text{s}$$

Find velocities

$$v = u + at$$

$$V_1 = 30 - 10(1.25) = 17.5\text{ms}^{-1}$$

$$V_2 = 10 - 10(1.25) = -2.5\text{ms}^{-1}$$

Part (iii)

We know when P_1 and P_2 at same height $t = 1.25\text{s}$.

Find time taken to reach max height for P_1

$$v = u + at$$

V is 0 at max height

$$0 = 30 - 10t \quad t = 3\text{s}$$

Time for P_1 above $P_2 = 3 - 1.25 = 1.75$ seconds

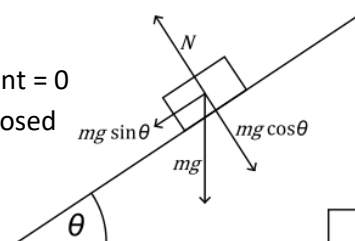
4. RESOLVING FORCES

- If force F makes an angle θ with a given direction, the effect of the force in that direction is $F \cos \theta$

$$F \cos(90 - \theta) = F \sin \theta$$

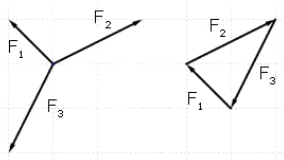
$$F \sin(90 - \theta) = F \cos \theta$$

- Forces in equilibrium: resultant = 0
- If drawn, forces will form a closed polygon



• **Methods of working out forces in equilibrium:**

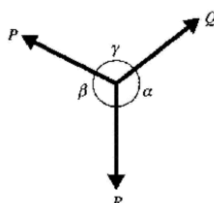
- Construct a triangle and work out forces
- Resolve forces in x and y directions; sum of each = 0



Lami's Theorem:

- For any set of three forces P, Q and R in equilibrium

$$\frac{P}{\sin \theta} = \frac{Q}{\sin \beta} = \frac{R}{\sin \gamma}$$



5. FRICTION

Friction = Coefficient of Friction $\times$ Normal Contact Force

$$F = \mu R$$

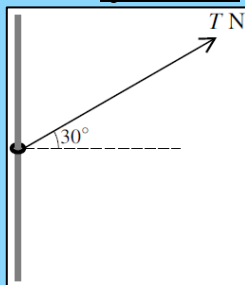
- Friction always acts in the opposite direction of motion
- Limiting equilibrium: on the point of moving, friction at max (limiting friction)
- Smooth contact: friction negligible
- **Contact force:**

- Refers to both F and N
- Horizontal component of Contact force = F
- Vertical component of Contact force = N
- Magnitude of Contact force given by the formula:

$$C = \sqrt{F^2 + N^2}$$

{W11-P43}

The ring has a mass of 2 kg . The horizontal rod is rough and the coefficient of friction between ring and rod is 0.24 . Find the two values of T for which the ring is in limiting equilibrium



Question 6:

Solution:

The ring is in limiting equilibrium in two different scenarios; we have to find T in both:

Scenario 1: ring is about to move upwards

Resultant = $T \sin 30 - \text{friction} - \text{Weight of Ring}$
Since the system is in equilibrium, resultant = 0:

$$\text{Contact Force} = T \cos 30$$

$$\therefore \text{Friction} = 0.24 \times T \cos 30$$

Substitute relevant information in to initial equation

$$0 = T \sin 30 - 0.24T \cos 30 - 20$$

$$T = 68.5\text{ N}$$

Scenario 2: ring is about to move downwards

This time friction acts in the opposite direction since friction opposes the direction of motion, thus:

$$\text{Resultant} = T \sin 30 + \text{Friction} - \text{Weight of Ring}$$

Using information from before:

$$0 = T \sin 30 + 0.24T \cos 30 - 20$$

$$T = 28.3\text{ N}$$

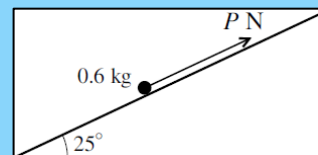
5.1 Equilibrium

Force required to keep a particle in equilibrium on a rough plane

Max Value	Min Value
MAX • The particle is about to move up • Thus friction force acts down the slope $P = F + mg \sin \theta$	MIN • The particle is about slip down • Thus frictional force acts up the slope $F + P = mg \sin \theta$

{W12-P43}

Question 6:



Coefficient of friction is 0.36 and the particle is in equilibrium. Find the possible values of P

Solution:

The magnitude of friction on particle in both scenarios is the same but acting in opposite directions

Calculate the magnitude of friction first:

$$\text{Contact Force} = 6 \cos 25$$

$$\therefore \text{Friction} = 0.36 \times 6 \cos 25$$

Scenario 1: particle is about to move upwards

$$P = 6 \sin 25 + \text{Friction}$$

$$P = 4.49\text{ N}$$

Scenario 2: particle is about to move downwards

$$P = 6 \sin 25 - \text{friction}$$

$$P = 0.578\text{ N}$$

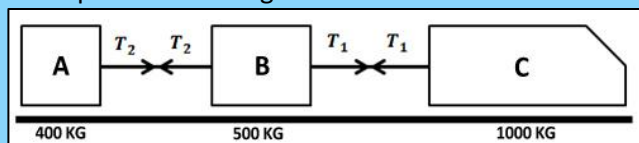
6. CONNECTED PARTICLES

Newton's 3rd Law of Motion:

For every action, there is an equal and opposite reaction

{Exemplar Question}

A train pulls two carriages:



The forward force of the engine is $F = 2500\text{N}$. Find the acceleration and tension in each coupling. The resistance to motion of A, B and C are 200, 150 and 90N respectively.

Solution:

To find acceleration, regard the system as a single object. The internal T 's cancel out and give:

$$2500 - (200 + 150 + 90) = 1900a$$

$$\therefore a = 1.08\text{ms}^{-2}$$

To find T_1 , look at C

$$F - T_1 - 200 = 1000a$$

$$2500 - T_1 - 200 = 1000 \times 1.08$$

$$T_1 = 1220\text{N}$$

To find T_2 , look at A

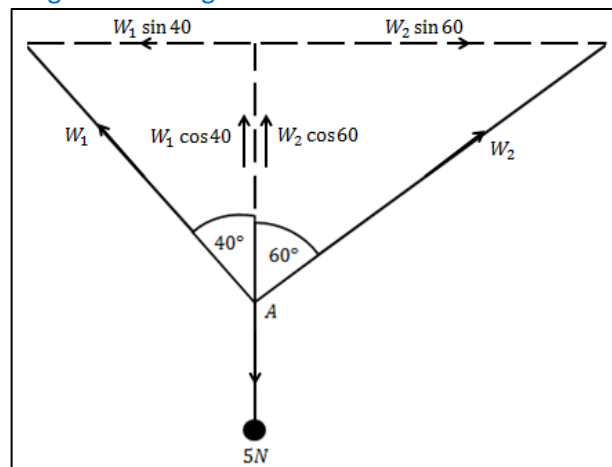
$$T_2 - 90 = 400a$$

$$T_2 - 90 = 400 \times 1.08$$

$$T_2 = 522\text{N}$$

Solution:

Diagram showing how to resolve forces:



Resolving forces at A vertically:

$$W_1 \cos 40 + W_2 \cos 60 = 5$$

Resolving forces at A horizontally:

$$W_1 \sin 40 = W_2 \sin 60$$

Substitute second equation into first:

$$\left(\frac{W_2 \sin 60}{\sin 40} \right) \cos 40 + W_2 \cos 60 = 5$$

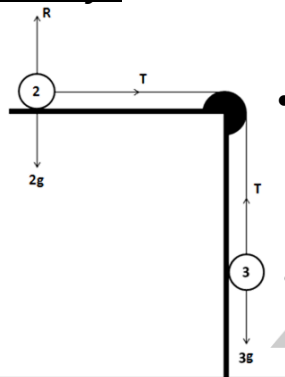
Solve to find W_2 :

$$W_2 = 3.26\text{N}$$

Put this value back into first equation to find W_1

$$W_1 = 4.40\text{N}$$

6.1 Pulleys

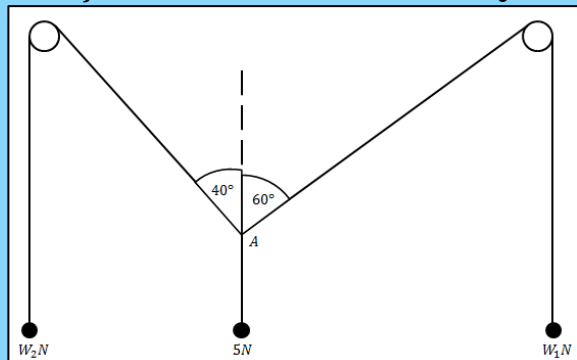


• Equation 1:
No backward force $\therefore$
 $T = 2a$

• Equation 2:
 $3g - T = 3a$

{W05-P04}

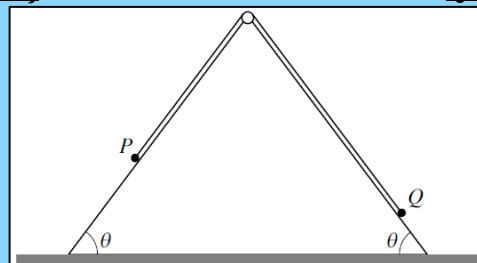
Question 3:



The strings are in equilibrium. The pegs are smooth. All the weights are vertical. Find W_1 and W_2

{S12-P41}

Question 6:



P has a mass of 0.6kg and Q has a mass of 0.4kg . The pulley and surface of both sides are smooth. The base of triangle is horizontal. It is given that $\sin \theta = 0.8$. Initially particles are held at rest on slopes with string taut. Particles are released and move along the slope

- Find tension in string. Find acceleration of particles while both are moving.
- Speed of P when it reaches the ground is 2ms^{-1} . When P reaches the ground it stops moving. Q continues moving up slope but does not reach the pulley. Given this, find the time when Q reaches its maximum height above ground since the instant it was released

Solution:**Part (i)**Effect of weight caused by P in direction of slope:Effect of weight = $mg \sin \theta$ where $\sin \theta = 0.8$ Effect of weight = $4.8N$ Effect of weight caused by Q in direction of slope:Effect of weight = $0.4 \times 10 \times 0.8 = 3.2N$ Body P has greater mass than body Q so when released P moves down Q moves up on their slopes $\therefore$

$$4.8 - T = 0.6a$$

$$T - 3.2 = 0.4a$$

Solve simultaneous equations:

$$\frac{4.8-T}{0.6} = \frac{T-3.2}{0.4} \quad T = 3.84N$$

Substitute back into initial equations to find a :

$$4.8 - 3.84 = 0.6a \quad a = 1.6ms^{-2}$$

Part (ii)Use kinematics equations to find the time which it take P to reach the ground:

$$a = \frac{v-u}{t} \text{ and } t = \frac{2-0}{1.6}$$

$$t_1 = 1.25s$$

When P reaches the ground, only force acting on Q is its own weight in the direction of slope = $3.2N$

$$F = ma \quad -3.2 = 0.4a$$

$$a = -8ms^{-2}$$

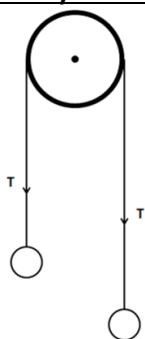
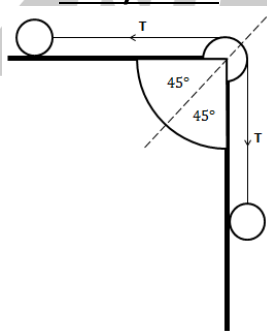
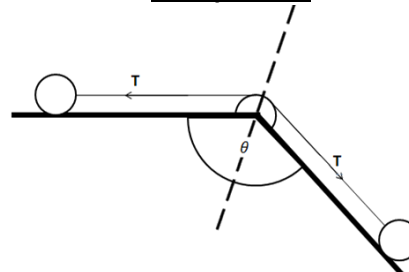
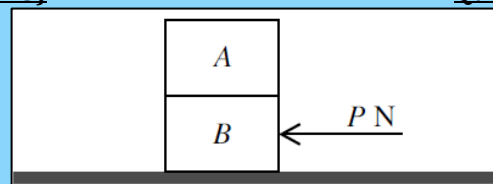
Now calculate the time taken for Q to reach max height

This occurs when its final velocity is 0.

$$-8 = \frac{0-2}{t} \quad t_2 = 0.25s$$

Now do simple addition to find total time:

$$\text{Total Time} = 1.25 + 0.25 = 1.5s$$

6.2 Force Exerted by String on Pulley**Pulley Case 1****Force on pulley = $2T$** **Acts: downwards****Pulley Case 2****Force on pulley = $T\sqrt{2}$** **Acts: along dotted line****Pulley Case 3****Force on pulley = $2T \cos\left(\frac{1}{2}\theta\right)$** **Acts: inwards along dotted line which bisects θ** **6.3 Two Particles****{S10-P43}****Question 7:**

A and B are rectangular boxes of identical sizes and are at rest on rough horizontal plane. A mass = $200kg$ and B mass = $250kg$. If $P \leq 3150$ boxes remains at rest. If $P > 3150$ boxes move

- Find coefficient of friction between B and floor
- Coefficient of friction between boxes is 0.2 . Given that $P > 3150$ and no sliding occurs between boxes. Show that the acceleration of boxes is not greater than $2ms^{-2}$
- Find the maximum possible value of P in the above scenario

Solution:**Part (i)**

$$F = \mu N$$

 F = to max P that does not move the boxes N = to contact force of **both** boxes acting on floor

$$\therefore 3150 = \mu \times (2000 + 2500)$$

$$\mu = 0.7$$

Part (ii)Find frictional force between A and B :

$$F = 0.2 \times 2000 \quad F = 400N$$

Use Newton's Second Law of Motion to find max acceleration for which boxes do not slide (below F)

$$400 = 200a \quad a = 2ms^{-2}$$

Part (iii)

P has to cause an acceleration of $2ms^{-2}$ on B which will pass on to A as they are connected bodies

Simply implement Newton's Second Law of Motion

$$\therefore P = (200 + 250)(2) + 3150$$

The 3150 comes from the force required to overcome the friction

$$P = 900 + 3150 \quad P = 4050N$$

7. WORK, ENERGY AND POWER

Principle of Conservation of Energy:

Energy cannot be created or destroyed, it can only be changed into other forms

Work Done: $W = Fs$

Kinetic Energy: $E_k = \frac{1}{2}mv^2$

Gravitational Potential Energy: $E_p = mgh$

Power: $P = \frac{W.d}{T}$ and $P = Fv$

7.1 Changes in Energy

$$\varepsilon_f - \varepsilon_i = (Work)_{engine} - (Work)_{friction}$$

- ε_f is the final energy of the object
- ε_i is the initial energy of the object
- $(Work)_{engine}$ is the energy caused by driving force acting on the object
- $(Work)_{friction}$ is the energy used up by frictional force or any resistive force

{S05-P04}

Question 7:

Car travelling on horizontal straight road, mass 1200kg. Power of car engine is 20kW and constant. Resistance to motion of car is 500N and constant. Car passes point A with speed $10ms^{-1}$. Car passes point B with speed $25ms^{-1}$. Car takes 30.5s to move from A to B.

- Find acceleration of the car at A
- Find distance AB by considering work & energy

Solution:

Part (i)

Use formula for power to find the force at A

$$P = Fv$$

$$20000 = 10F \quad \text{Driving force} = 2000N$$

We must take into account the resistance to motion

$$\therefore F = \text{Driving Force} - \text{Resistance} = 2000 - 500$$

$$F = 1500$$

Use Newton's Second Law to find acceleration:

$$1500 = 1200a \quad a = \frac{1500}{1200} = 1.25ms^{-2}$$

Part (ii)

Use power formula to find work done by engine:

$$P = \frac{w.d.}{t}$$

$$20000 = \frac{w.d.}{30.5} \quad w.d. = 610000J$$

There is change in kinetic energy of the car so that means some work done by the engine was due to this:

$$k.E. \text{ at A} = \frac{1}{2}1200(10)^2 \quad k.E. \text{ at B} = \frac{1}{2}1200(25)^2$$

$$\text{Change in } k.E. = k.E. \text{ at B} - k.E. \text{ at A}$$

$$\text{Change in } k.E. = 375000 - 60000 = 315000$$

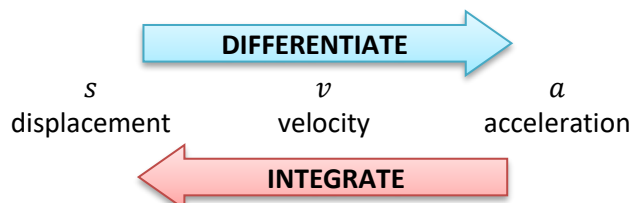
There is also some work done against resistive force of 500N; due to law of conservation of energy, this leads us to the main equation:

$$w.d. \text{ by engine} = \text{change in } k.E. + w.d. \text{ against resistance}$$

$$610000 = 315000 + 500s$$

$$s = \frac{610000 - 315000}{500} = \frac{295000}{500} = 590m$$

8. GENERAL MOTION IN A STRAIGHT LINE



- Particle at instantaneous rest, $v = 0$
- Maximum displacement from origin, $v = 0$
- Maximum velocity, $a = 0$

{W10-P42}

Question 7:

Particle P travels in straight line. It passes point O with velocity $5ms^{-1}$ at time $t = 0s$.

P's velocity after leaving O given by:

$$v = 0.002t^3 - 0.12t^2 + 1.8t + 5$$

v of P is increasing when: $0 < t < T_1$ and $t > T_2$

v of P is decreasing when: $T_1 < t < T_2$

- Find the values of T_1 and T_2 and distance OP when $t = T_2$
- Find v of P when $t = T_2$ and sketch velocity-time graph for the motion of P

Solution:

Part (i)

Find stationary points of v ; maximum is where $t = T_1$ and minimum is where $t = T_2$

$$\frac{dv}{dt} = 0.006t^2 - 0.24t + 1.8$$

Stationary points occur where $\frac{dv}{dt} = 0$

$$\therefore 0.006t^2 - 0.24t + 1.8 = 0$$

Solve for t in simple quadratic fashion:

$$t = 30 \quad \text{and} \quad 10$$

Naturally T_1 comes before T_2

$$\therefore T_1 = 10s \quad \text{and} \quad T_2 = 30$$

Finding distance OP by integrating

$$\therefore s = \int_0^{30} v \, dt$$

$$s = \int_0^{30} (0.002t^3 - 0.12t^2 + 1.8t + 5) \, dt$$

$$s = \left[0.0005t^4 - 0.04t^3 + 0.9t^2 + 5t \right]_0^{30}$$

$$s = 285m$$

Part (ii)Do basic substitution to find v

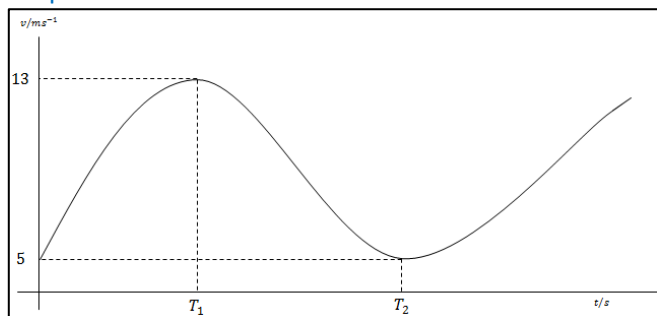
$$v = 0.002t^3 - 0.12t^2 + 1.8t + 5$$

$$t = 30 \quad v = 5$$

To draw graph find v of P at T_1 using substitution and plot roughly

$$v \text{ at } T_1 = 13$$

Graph:



Since the distance before 20 seconds has already been taken into consideration:

$$t = t - 20$$

$$a = \frac{6.5 - 2}{6}$$

$$a = 0.75$$

$$\therefore s_2 = 2(t - 20) + \frac{1}{2}(0.75)(t - 20)^2$$

$$s_2 = 2t - 40 + 150 + 0.375t^2 - 15t$$

$$s_2 = 0.375t^2 - 13t + 110$$

Finally add both to give you s

$$s = s_1 + s_2$$

$$s = 0.375t^2 - 13t + 110 + 92$$

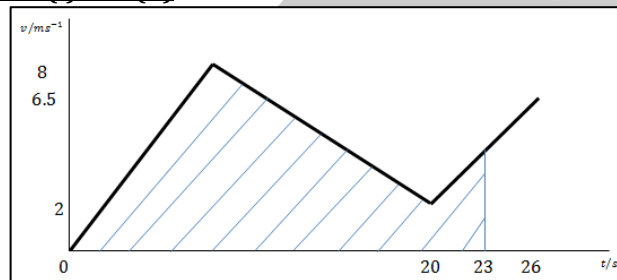
$$s = 0.375t^2 - 13t + 202$$

{S13-P42}**Question 6:**

Particle P moves in a straight line. Starts at rest at point O and moves towards a point A on the line. During first 8 seconds, P 's speed increases to 8ms^{-1} with constant acceleration. During next 12 seconds P 's speed decreases to 2ms^{-1} with constant deceleration. P then moves with constant acceleration for 6 seconds reaching point A with speed 6.5ms^{-1}

- Sketch velocity-time graph for P 's motion
- The displacement of P from O , at time t seconds after P leaves O , is s metres. Shade region of the velocity-time graph representing s for a value of t where $20 \leq t \leq 26$
- Show that for $20 \leq t \leq 26$,

$$s = 0.375t^2 - 13t + 202$$

Solution:**Part (i) and (ii)****Part (ii)**First find s when $t = 20$, this will produce a constant since $20 \leq t \leq 26$

$$s_1 = \frac{1}{2}(8)(8) + \frac{1}{2}(8 + 2)(12) = 92\text{m}$$

Finding s when $20 \leq t \leq 26$:

$$s = ut + \frac{1}{2}at^2$$

CIE A-LEVEL MATHEMATICS//9709



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